[2] Braslow, A. L., "Review of the effect of distributed surface roughness on boundary-layer transition," AGARD Rept. 254 (April 1960).

[3] van Driest, E. R. and Blumer, C. B., "Boundary-layer transition at supersonic speeds—three-dimensional roughness effects (spheres)," J. Aerospace Sci. 29, 909–916 (1962).

Correlation of Laminar Heating to Cones in High-Speed Flight at Zero Angle of Attack

Richard L. Schapker*
Avco-Everett Research Laboratory, Everett, Mass.

Laminar heating to pointed cones in high-speed flight at zero angle of attack is correlated by a simple expression involving only freestream density, velocity, and cone angle. Comparison of the theory with experiments and a flat-plate laminar heating correlation is very satisfactory. It is found that the ratio of (dimensionless) wall enthalpy gradient parameters $(\partial g/\partial\eta)_w$ for cones and axisymmetric stagnation points is very nearly equal to unity.

Nomenclature

g = enthalpy ratio, H/H_e
g' = wall enthalpy gradient, $\partial g/\partial\eta$
H = total enthalpy
h = static enthalpy
p = static pressure
Pr = Prandtl number
q = heating rate, Btu-sec^{-1}-ft^{-2}
Q = integrated heating rate, Btu-sec^{-1}
r = distance from axis normal to U_∞, ft
R = blunt-body nose radius, ft
T = temperature
U = velocity, ft-sec^{-1}
x = axial distance, ft
x' = distance along body surface, ft
δ = cone half-angle
μ = absolute viscosity, slugs-(ft-sec)$^{-1}$
ξ = transformation variable, slugs^2-ft^{-2}
ρ = density, slugs-ft^{-3}

Subscripts

c = cone; external to cone boundary layer
e = external to boundary layer
s = blunt-body stagnation point
sl = sea level
w = wall
∞ = freestream

Certain aspects of re-entry analysis require knowledge of aerodynamic heating, and so it is desirable to have a means of determining this quantity in a simple, accurate manner for various body shapes. Stagnation heating to axisymmetric bodies already has been correlated satisfactorily as[1]

$$q_s R^{1/2} = 867(\rho_\infty/\rho_{sl})^{1/2}(U_\infty/10^4)^{3.15} \times [(H_e - h_w)/(H_e - h_{w_{300°K}})] \quad (1)$$

which agrees with detailed calculations (and experiments[1]) to within ± 10% for all altitudes up to 250 kft and velocities between 7 and 25 kft/sec. It is the purpose herein to present a similar relation for laminar heating to cones in hypersonic flight at zero angle of attack.

In light of the existence of Eq. (1), heating to cones is known if the ratio of cone to stagnation point heating can be evaluated. To this end, consider the case of Lewis number equal to 1, whereby laminar heating is given as[2]

$$q = (r\rho\mu g')_w U_e H_e/Pr(2\xi)^{1/2} \quad (2)$$

Assuming constant wall properties, the independent variable ξ is[2]

$$\xi_s = (\rho_w\mu_w)_s(x')^4(dU_e/dx')_s/4 \quad (3)$$

in the stagnation point region of blunt axisymmetric bodies, and

$$\xi_c = (\rho_w\mu_w U x^3)_c \tan^2\delta/3\cos\delta \quad (4)$$

for cones. If H_e and T_w are the same in both cases, and $\mu = \mu(T)$, then the ratio of cone to axisymmetric stagnation heating follows from Eqs. (2–4) as

$$\frac{q_c}{q_s} = \frac{3^{1/2}}{2} G\left[\frac{\cos\delta}{x_c}\left(\frac{\rho_c}{\rho_s}\right)_w \frac{U_c}{(dU_e/dx')_s}\right]^{1/2} \quad (5)$$

where G is the ratio of wall enthalpy gradient parameters $(g_c'/g_s')_w$.

The density ratio $(\rho_c/\rho_s)_w$ is equal to the pressure ratio p_c/p_s, since wall temperatures are the same. In accordance with the Newtonian flow approximation,

$$p/p_\infty = 1 + \gamma_\infty(M_\infty \sin\delta)^2$$

where $\delta = \pi/2$ for a stagnation point. Therefore,

$$(\rho_c/\rho_s)_w = (p_c/p_\infty)/(p_s/p_\infty) \simeq 1/\gamma_\infty M_\infty^2 + \sin^2\delta \simeq \sin^2\delta \quad (6)$$

for $M_\infty \simeq 20$ and $\delta > 5°$.

The velocity external to the cone boundary layer U_c is given by the hypersonic approximation

$$U_c = U_\infty \cos\delta \quad (7)$$

Combining Eqs. (5–7) and using the velocity derivative $(dU_e/dx')_s$ corresponding to Newtonian flow in the stagnation region,[3] one obtains

$$q_c\, x_c^{1/2}/q_s\, R^{1/2} = 0.364\, G \sin 2\delta(\rho_s/\rho_\infty)^{1/4} \quad (8)$$

as the cone-stagnation point heating ratio.

The stagnation density ratio ρ_s/ρ_∞ is given by Feldman[4] over the velocity range $8 \leq U_\infty \leq 24$ kft/sec for altitudes up to 250 kft. His data are correlated by

$$\rho_s/\rho_\infty = 8.4(U_\infty/10^4)^{0.8} \quad (9)$$

the one-fourth power of which is accurate to within 6% for all velocities within this range and altitudes greater than 50 kft.

Therefore, inserting Eq. (9) into (8) and using (1), heating to cones in high-speed flight at zero angle of attack is given as (leaving out the wall enthalpy correction)

$$q_c\, x_c^{1/2} = 535\, G \sin 2\delta(\rho_\infty/\rho_{sl})^{1/2}(U_\infty/10^4)^{3.35} \quad (10)$$

The total heat input Q follows by integrating Eq. (10) over the body surface (exclusive of the base):

$$Q = 4500\, GL^{3/2} \sin\delta \tan\delta(\rho_\infty/\rho_{sl})^{1/2}(U_\infty/10^4)^{3.35} \quad (11)$$

To complete the correlation, Eq. (10), it remains to specify the ratio of wall enthalpy gradient parameters G.

A result of the calculations of Kemp et al.[3] was that the wall enthalpy gradient parameter $g_w'/(1 - g_w)$ is practically unaffected by the level of the dissipation parameter U_e^2/H_e for a given value of pressure gradient parameter β. This means that the difference between g_{wc}' and g_{ws}' arises primarily because $\beta = 0$ for a cone and $\frac{1}{2}$ for a (spherical) stag-

Received June 3, 1963. This work was sponsored by Bell Telephone Laboratories under Army Missile Command Prime Contract DA-30-069-ORD-1955. The author wishes to thank Nelson Kemp of Avco-Everett Research Laboratory for his helpful discussions during this work.

* Associate Engineer. Member AIAA.

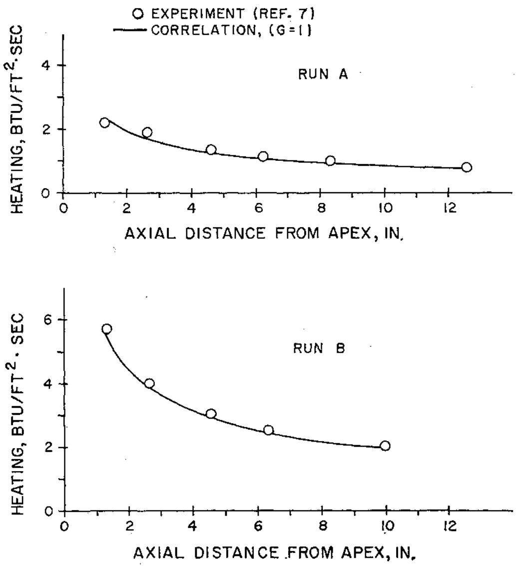

Fig. 1.

nation point and is essentially independent of the value of $\rho_e\mu_e/\rho_w\mu_w$ for the two bodies. In light of this result, then, one obtains G from the correlation given in Fig. 1 of Ref. 3:

$$(g_c'/g_s')_w = G = [1 + 0.096(\tfrac{1}{2})^{1/2}]^{-1} = 0.936 \qquad (12)$$

However, upon closer examination, Fig. 1 of Ref. 3 shows that the fluid property plus dissipation term modifications move the points lying on the $\beta = 0$ curve sufficiently upward and to the right such that, in comparing the stagnation $(U_e^2/H_e = 0)$ to the cone $(U_e^2/H_e \rightarrow 2)$ solutions, one is quite justified in taking G equal to 1. In fact, it will be seen that the few experimental data available imply a value for G slightly greater than one.

Recently, Hanley[5] published a correlation of Cohen's numerical results[6] for laminar heating to flat plates at high speed in the form

$$q = 1.17(10^4)\left(\frac{1}{x}\frac{U_e}{U_\infty}\frac{p_e}{p_\infty}\rho_\infty\right)^{1/2}\left(\frac{T_w}{900}\right)^{-0.051} \times \left(\frac{U_\infty}{10^4}\right)^{3.21}\left(1 - 1.205\frac{h_w}{H_e}\right) \qquad (13)$$

Since heating to cones is greater by a factor $3^{1/2}$ than that to flat plates, a correlation in Hanley's form can be obtained immediately from Eqs. (1, 5, and 9) as

$$\frac{q_c}{3^{1/2}} = 1.272(10)^4 G\left(\frac{1}{x_c'}\frac{U_c}{U_\infty}\frac{p_c}{p_s}\rho_\infty\right)^{1/2}\left(\frac{U_\infty}{10^4}\right)^{3.35} \qquad (14)$$

Besides a small difference in velocity dependence of 3.21 instead of 3.35, Hanley's constant implies a value for G of 0.92, which is close to that from Kemp et al. [cf., Eq. (12)]. One concludes that the calculations of Cohen and Kemp et al. are in very good agreement if one assumes that $g_w'/(1 - g_w)$ is independent of $\rho_e\mu_e/\rho_w\mu_w$.

The scarcity of heating data for cones in hypersonic flight at zero angle of attack allows only tentative confirmation of the theory. Figure 1, using the data of Ref. 7, shows two runs made at the conditions shown in Table 1.

The data are seen to be correlated very well by Eq. (10) with $G = 1$, although the consistently lower heating predicted by this relation indicates that a value of G slightly greater than 1 perhaps may be even better. However, no definite conclusions can be drawn until more data are available, and so using $G = 1$ should give accurate predictions of laminar heating to cones.

Table 1 Test conditions

Run	M_∞	H_∞, ft²/sec²	$\rho_\infty U_\infty^2/2$, psi	ρ_∞/ρ_{sl}	δ
A	15.89	27.3(10⁶)	0.094	2.09(10⁻⁴)	8.3°
B	16.77	28.7(10⁶)	0.435	9.2(10⁻⁴)	8.3°

The variable ξ_c [cf., Eq. (4)] was determined under the assumption of a pointed cone, whereas in reality re-entering cones will become blunted to some extent. Lees[8] has shown, however, that blunting has a minor effect on the conical heating when the length is measured from the virtual apex of the blunted cone. This follows from the assumption of local similarity, which neglects flow history up to any point, and from the fact that the pressure on the conical portion is the same (within the accuracy of the Newtonian approximation) whether a cone is blunted or not. Hence, Eq. (10) (along with the wall enthalpy correction) can be used over any laminar portion of a trajectory with $G = 1$.

References

[1] Detra, R. W., Kemp, N. H., and Riddell, F. R., "Addendum to 'Heat transfer to satellite vehicles re-entering the atmosphere,'" Jet Propulsion **27**, 1256–1257 (1957).

[2] Fay, J. A. and Riddell, F. R., "Theory of stagnation point heat transfer in dissociated air," J. Aerospace Sci. **25**, 73–85 (1958).

[3] Kemp, N. H., Rose, P. H., and Detra, R. W., "Laminar heat transfer around blunt bodies in dissociated air," J. Aerospace Sci. **26**, 421–430 (1959).

[4] Feldman, S., "Hypersonic gas dynamic charts for equilibrium air," Avco-Everett Research Lab. Res. Rept. 40 (1957).

[5] Hanley, G. M., "Hypervelocity laminar convective flat plate heating," ARS J. **32**, 1740–1743 (1962).

[6] Cohen, N. B., "Boundary layer similar solutions and correlation equations for laminar heat transfer distribution in equilibrium air at velocities up to 41,000 feet per second," NASA TR R-118 (1961).

[7] Tolosko, R. J., private communication, Avco Res. and Advanced Dev. Div. (March 1963).

[8] Lees, L., "Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds," Jet Propulsion **26**, 259 (1956).

Approximate Analysis of Propellant Stratification

Daniel M. Tellep* and Edward Y. Harper†
Lockheed Missiles and Space Company, Sunnyvale, Calif.

Introduction

STRATIFICATION refers to the development of temperature gradients in a fluid container subjected to external heating. A free-convection boundary layer carries the heated fluid at the walls to the top of the tank, forming a growing layer of "stratified" liquid which is at a higher temperature than the bulk. Thus, ullage pressure rise of a saturated fluid in a closed container is greater than the pressure rise that would occur if the fluid were mixed at a uniform temperature.

The system analyzed is a closed cylindrical tank accelerating along its longitudinal axis and filled to some height H with liquid, which is subjected to a uniform side wall heat flux $\dot{q}$. The ullage is assumed to be at the saturation pressure corresponding to the liquid surface temperature. Wall heat flux to the ullage is considered to be negligible. At time zero, a steady turbulent wall boundary layer is formed instantaneously in the liquid, which is at a constant bulk temperature T_b. The unknowns of the problem are the surface temperature T_s, the ullage pressure P_u, and the stratified layer thickness Δ.

An approximate solution is obtained by assuming a di-

Received June 12, 1963.

* Manager, Launch and Entry Thermodynamics. Associate Member AIAA.

† Thermodynamics Engineer.